

## Analysis of piston behavior according to eccentricity ratio of disk in bent-axis type piston pump

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### Abstract

To improve the performance of the bent-axis type axial piston pump driven by the tapered piston, it is necessary to know the driving characteristics and mechanism of the tapered piston and the cylinder block. Since each piston not only rotates on its axis and reciprocates in the cylinder bore but also revolves around the axis of the driving shaft, it is difficult to analyze the driving mechanism theoretically. The theoretical mechanism for the bent-axis type axial piston pump is studied by using the geometrical method. The driving range of the tapered piston is determined by theoretical equations. The results show that the cylinder block is driven by one tapered piston in a limited range and the core parameters such as the tilting angle of the piston and the ahead delay angle influence performance of the bent-axis type axial piston pump.

*Keywords:* Hydraulic piston pump; Tapered piston; Swivel angle; Delay angle; Eccentricity ratio

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### 1. Introduction

The oil hydraulic system used in large and heavy equipment has been modified constantly because of its limited performance, negative environmental influence, and noise. To solve these problems, research has been carried out in the areas of size reduction of oil hydraulic systems, high speed and pressure, electronic control, substitute oil, noise decrease, and etc.

The bent-axis type oil hydraulic piston pump acting as the core power source in the oil hydraulic system is no exception to this technique tendency. It is used as the main pump in heavy construction equipment because it offers high speed and pressure, high total efficiency and distinguished variable delivery.

Related researches in this field are extremely limited. The cylinder block in the rotary part is driven by a tapered piston connected to the disk of the shaft and the geometrical mechanism of this type of pump is very complicated and difficult to analyze.

Such scarce but valuable research that has been carried out as mentioned above focuses on the performance of the axial piston pump [1], the driving mechanism of the axial piston pump [2], the friction loss of the axial piston pump [3] and so on.

The driving mechanism of the bent-axis type piston pump driven by the piston rod is the focus of this study. The piston rod drives the cylinder block, so the taper angle of the piston rod and the swivel angle between the cylinder block and the shaft are very important design factors. If the above design factors do not satisfy the optimized conditions, the friction loss between the cylinder bore and the piston rod

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would increase, and the pump could even fail to run in conditions with severe friction and wear.

When the piston rod reciprocates in the cylinder bore with high velocity, it rotates on its own axis and revolves about the center line of the cylinder block, and the driving force produced in the contacted part between the tapered piston rod and the inner surface of the cylinder bore drives the cylinder block simultaneously. Because of all those complicated modes of motion, it is not easy to analyze the pump’s driving mechanisms.

Moreover, in the manufacturing processes of the disk, the cylinder block and the piston, the tolerances related to the design drawing and the implements take place. Accordingly, considering these tolerances, the driving mechanism becomes very complicated and it is almost impossible to analyze. In this study, the driving characteristic according to the changes of the position tolerance of the disk’s spherical part which undergoes the difficulty of the tolerance management in oil hydraulic maker is determined.

Therefore, in this paper, the driving mechanism of the piston rod in the bent-axis type piston pump driven by the piston rod has been grasped; and, when the center of the disk is eccentric, a theoretical analysis is performed of the influences of eccentric parameters on the delay angle of the piston rod and the driving ranges of the piston.

## 2. Geometrical mechanism of piston rod

### 2.1 Under condition without disk eccentricity

Fig. 1 illustrates the function principle of the piston pump. When the driving shaft is driven by a motor (an electronic motor, an engine, etc.), the head of the piston rod connected to the center of the spherical part in the disk rotates on the axis of the shaft. On the

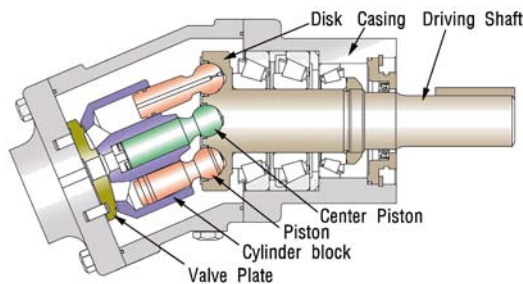


Fig. 1. Diagram of axial piston pump.

other hand, the end of the piston rod connects with the cylinder bore, and the consequent lateral force acted on contacted part of the cylinder bore and the piston rod drives the cylinder block to rotate about the center line of the cylinder block. The piston rod reciprocates in the cylinder bore, and at the same time, it rotates on its own axis and revolves on the center of the cylinder block simultaneously.

Because of the swivel angle between the cylinder block and the driving shaft, as the shaft rotates, the piston moves upwards to TDC and working oil flows in from the suction port in the valve plate to the cylinder block. Then the piston moves downwards to BDC and working oil flows out from the cylinder block to the discharge port.

Fig. 2 shows the driving mechanism of the piston rod of the bent-axis type piston pump. From the viewpoint of the cylinder block, the circle of the spherical part of the disk with radius of  $R_d$  is seen as an ellipse and  $R_{dc}$  is expressed by the following equation:

$$R_{dc} = R_d \sqrt{\sin^2 \theta + (\cos \theta \cos \alpha)^2} \tag{1}$$

Fig. 3 shows the diagram of the phase angle and the tilting angle related with the behavior of the piston head.

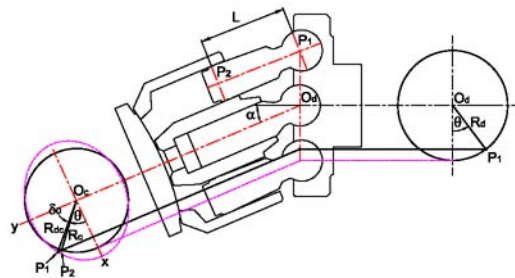


Fig. 2. Driving mechanism of axial piston pump.

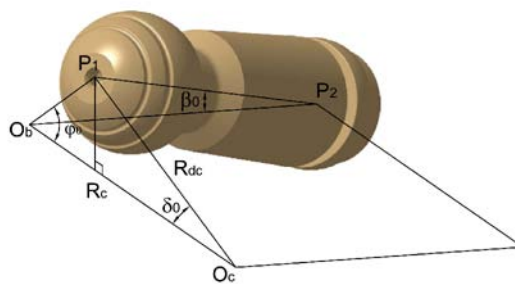


Fig. 3. Diagram of phase angle and tilting angle.

With the shaft rotating, the ahead behavior of the center of the piston head ( $P_1$ ) results in the angle differences between the center of the cylinder block ( $O_c$ ) and the center of the cylinder bore ( $O_b$ ). The two angles are the so-called ahead delay angle and the phase angle.

The corresponding equations of the ahead delay angle ( $\delta_0$ ) in four quadrants are listed:

$$0 \leq \theta < \frac{\pi}{2}$$

$$\delta_0 = \sin^{-1} \left( \frac{R_d \sin \theta}{R_{dc}} \right) - \theta \quad (2)$$

$$\frac{\pi}{2} \leq \theta < \pi$$

$$\delta_0 = -\sin^{-1} \left( \frac{R_d \sin \theta}{R_{dc}} \right) - \theta + \pi \quad (3)$$

$$\pi \leq \theta < \frac{3\pi}{2}$$

$$\delta_0 = -\sin^{-1} \left( \frac{R_d \sin \theta}{R_{dc}} \right) - \theta + \pi \quad (4)$$

$$\frac{3\pi}{2} \leq \theta \leq 2\pi$$

$$\delta_0 = \sin^{-1} \left( \frac{R_d \sin \theta}{R_{dc}} \right) - \theta + 2\pi \quad (5)$$

and then, the phase angle is given as follows:

$$\varphi_0 = \sin^{-1} \left( \frac{R_{dc} \sin \delta_0}{\sqrt{(R_c - R_{dc} \cos \delta_0)^2 + (R_{dc} \sin \delta_0)^2}} \right) \quad (6)$$

The tilting angle  $\beta_0$  is the angle difference between the center line of piston ( $\overline{P_1P_2}$ ) and the center line of piston bore ( $\overline{O_bP_2}$ )

$$\beta_0 = \sin^{-1} \frac{\sqrt{R_c^2 + R_{dc}^2 - 2R_cR_{dc} \cos \delta_0}}{L} \quad (7)$$

Fig. 4 shows the delay angle generated between the piston rod and the cylinder block. When the rotating angle of the shaft is  $\theta$ , from the viewpoint of the cylinder block, the angle between  $\overline{P_1O_c}$  and  $\overline{P_1O_b}$  is the delay angle ( $\delta_x$ ).

$$\delta_x = \cos^{-1} \frac{R_c^2 + R_{dc}^2 - (L \sin \xi)^2}{2R_cR_{dc}} - \delta_0 \quad (8)$$

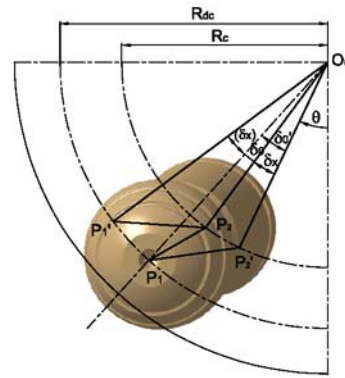


Fig. 4. Diagram of delay angle.

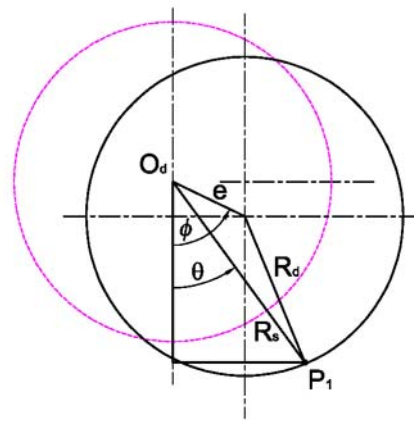


Fig. 5. Eccentric distance and angle on disk of shaft.

### 2.2 Under condition with disk eccentricity

Fig. 5 shows a plan sketch of the piston rod when the disk is eccentric with a certain degree  $\phi$  and distance  $e$ . Here, the eccentric distance  $e$  is the product of the eccentricity ratio  $e'$  and the radius of the disk's spherical part. Under this condition, from the viewpoint of the disk, the distance ( $R_s$ ) between  $P_1$  and  $O_d$  is given as follows:

$$R_s = e'R_d \cos(\phi - \theta) + \sqrt{R_d^2 - [e \sin(\phi - \theta)]^2} \quad (9)$$

Here,  $R_e$  is equivalent with  $R_{dc}$ , which is expressed above in Eq. (1). It changes according to the eccentric angle  $\phi$  and eccentric distance  $e$ , and the equation is shown as follows:

$$R_e = R_s \sqrt{\sin^2 \theta + (\cos \alpha \cos \theta)^2} \quad (10)$$

Then, the equivalent ahead delay angle ( $\delta_e$ ) varies with the rotating angle, and the equations in the different quadrants are:

$$0 \leq \theta < \frac{\pi}{2}$$

$$\delta_e = \sin^{-1} \left( \frac{\left( e' R_d \cos(\phi - \theta) + \sqrt{R_d^2 - [e' R_d \sin(\phi - \theta)]^2} \right) \sin \theta}{R_c} \right) - \theta \quad (11)$$

$$\frac{\pi}{2} \leq \theta < \pi$$

$$\delta_e = -\sin^{-1} \left( \frac{\left( e' R_d \cos(\phi - \theta) + \sqrt{R_d^2 - [e' R_d \sin(\phi - \theta)]^2} \right) \sin \theta}{R_c} \right) - \theta + \pi \quad (12)$$

$$\pi \leq \theta < \frac{3\pi}{2}$$

$$\delta_e = -\sin^{-1} \left( \frac{\left( e' R_d \cos(\phi - \theta) + \sqrt{R_d^2 - [e' R_d \sin(\phi - \theta)]^2} \right) \sin \theta}{R_c} \right) - \theta + \pi \quad (13)$$

$$\frac{3\pi}{2} \leq \theta \leq 2\pi$$

$$\delta_e = \sin^{-1} \left( \frac{\left( e' R_d \cos(\phi - \theta) + \sqrt{R_d^2 - [e' R_d \sin(\phi - \theta)]^2} \right) \sin \theta}{R_c} \right) - \theta + 2\pi \quad (14)$$

The equivalent phase angle ( $\varphi_e$ ), the equivalent tilting angle ( $\beta_e$ ), and the equivalent ahead delay angle ( $\delta_e$ ) are also worked out one by one:

$$\varphi_e = \sin^{-1} \left( \frac{R_e \sin \delta_e}{\sqrt{(R_c \sin \delta_e)^2 + (R_c - R_e \cos \delta_e)^2}} \right) \quad (15)$$

$$\beta_e = \sin^{-1} \frac{\sqrt{R_c^2 + R_e^2 - 2R_c R_e \cos \delta_e}}{L} \quad (16)$$

$$\delta_E = \cos^{-1} \frac{R_c^2 + R_e^2 - (L \sin \xi)^2}{2R_c R_e} - \delta_e \quad (17)$$

The main parameters needed in the analysis of the rotary part in the bent-axis type piston pump driven by the piston rod are arranged in Table 1.

Table 1. Geometric data of rotary part.

$e'$	0~20E-4	-
$\alpha$	5~30	[deg]
$L$	62	[mm]
$R_d$	73.5	[mm]
$R_c$	69.5	[mm]
$\xi$	1.9	[deg]

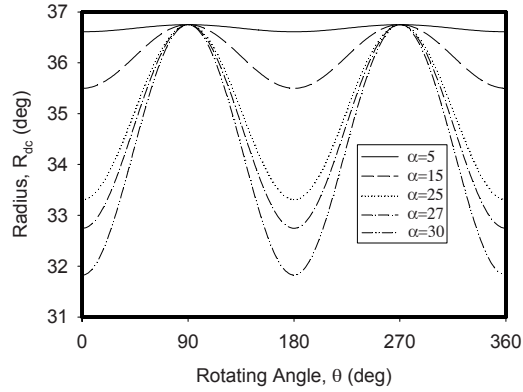


Fig. 6. Radius  $R_{dc}$  to swivel angle.

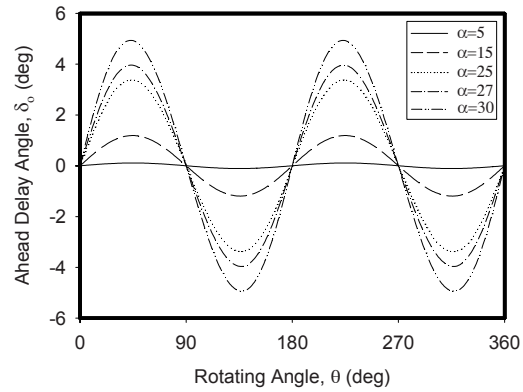


Fig. 7. Ahead delay angle to swivel angle.

### 3. Theoretical results

#### 3.1 Under condition without disk eccentricity

Fig. 6 shows the influence on  $R_{dc}$  when the swivel angle is 5, 15, 25, 27, and 30deg, respectively. When the swivel angle increases, the piston stroke becomes larger; consequently, the difference between the maximum and minimum of  $R_{dc}$  increases.

Fig. 7 shows the influence of the increase of the swivel angle on the ahead delay angle. Because the bent-axis type piston pump is driven by the tapered

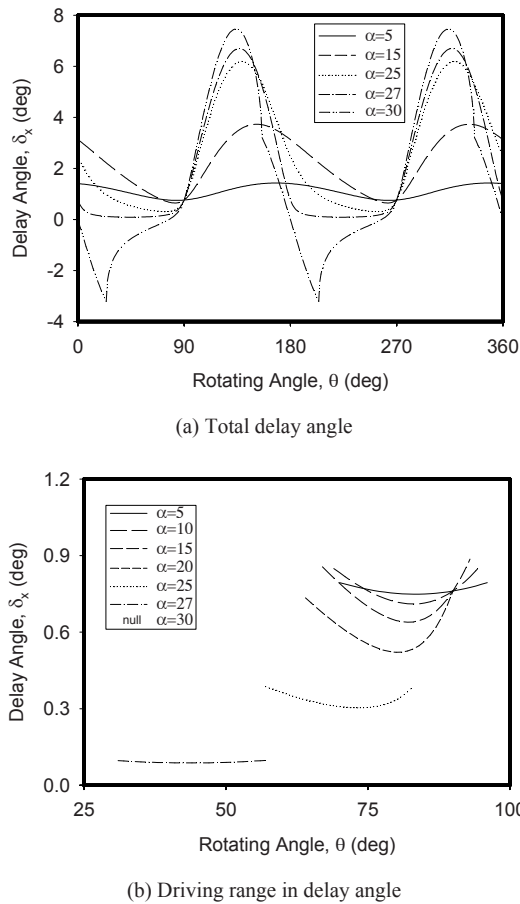


Fig. 8. Delay angle to swivel angle.

piston rod and the ahead delay angle exists, it is impossible to drive the piston rod and the cylinder block simultaneously. That is, when the driving shaft rotates, the head of the piston rod connected with the center of the spherical part in the disk revolves on the axis of the shaft, the end of the piston rod connects with the cylinder bore, and the consequent lateral force acted on the contacted part of the inner surface of the cylinder bore and the piston rod drives the cylinder block to rotate about the center line of the cylinder block. By the above principle, as the swivel angle is the larger, the rate of increase of the ahead delay angle is also greater.

Fig. 8 shows the influence of the delay angle with the change of the swivel angle.

In the bent-axis type piston pump driven by the piston rod, because the delay angle can distinguish the possibility of normal driving, it is generally used as a criterion for the optimum design of the swivel angle

and the taper angle. A single piston rod performs the driving action on the cylinder block twice at one revolution of the shaft [2].

Based on the assumption that the piston has seven piston rods, the driving range of one piston rod with the minimum delay angle is about 25.7 deg, and there are two driving ranges for one rotation of the piston rod.

Fig. 8 shows that the driving ranges where piston rod makes the cylinder bore rotate mainly exist in the first and the third quadrants. When the rotating angle passes by 45 deg and 225 deg, the distance between the head of the piston and the center of the cylinder bore becomes shorter and the ahead delay angle decreases, so the driving ranges with the minimum delay angle are generated

In addition, the minimum delay angle gradually approaches zero with the increase of the swivel angle, and when the swivel angle is above 27 deg, the minimum delay angle becomes less than zero, so it is impossible to drive normally.

### 3.2 Under condition with disk eccentricity

Considering Fig. 8, it is noticed that the delay angle changes greatly even for a slight change of the swivel angle. So in the real design process, the geometrical tolerances will consequently deteriorate the performance because of the change of the delay angle. Because the delay angle is the focus in this paper, the influence of the disk eccentricity from the center of the shaft which is the main factor determining geometrical tolerances on the delay angle has to be understood.

Figs. 9 to 13 show the change of the equivalent delay angle according to the eccentricity ratio of the disk from the center of the shaft (where  $\alpha = 5, 15, 25, 27, 30$  deg).

Fig. 9 shows the change of the delay angle according to the change of the eccentricity ratio for  $\alpha = 5$  deg. The minimum delay angle approaches zero when the eccentricity ratio increases. Assuming that there are seven piston rods, the driving range of one piston rod with the minimum delay angle is about 25.7 deg, and it can be confirmed that there are two driving ranges at one rotation of the piston rod. When the eccentricity ratio is  $20E-4$ , the minimum delay angle decreases below zero, and the piston rod cannot drive the cylinder block normally.

Fig. 10 and Fig. 11 show the changes of the delay

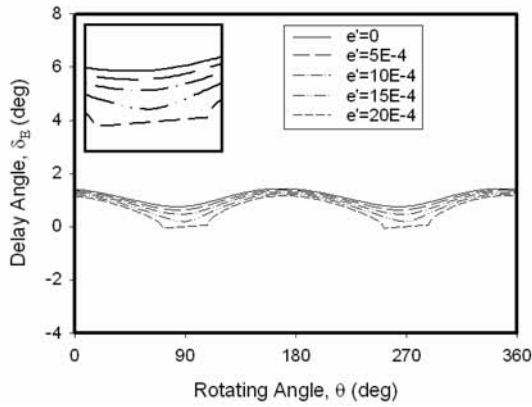


Fig. 9. Delay angle to eccentric ratio ( $\alpha=5$ deg).

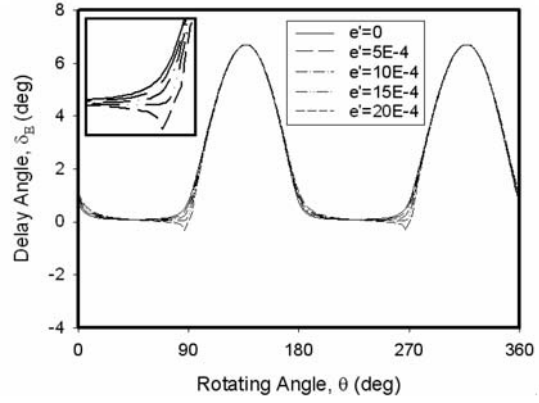


Fig. 12. Delay angle to eccentric ratio ( $\alpha=27$ deg).

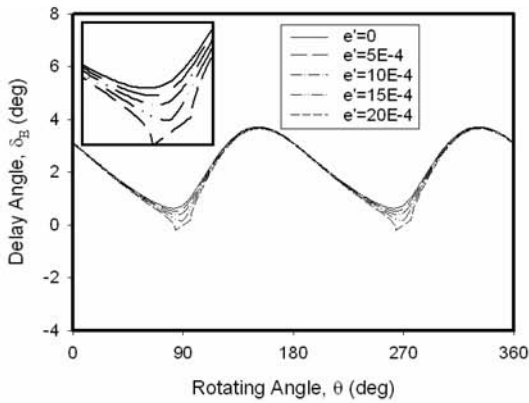


Fig. 10. Delay angle to eccentric ratio ( $\alpha=15$ deg).

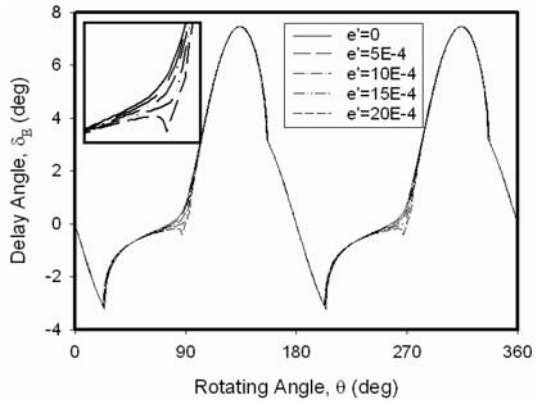


Fig. 13. Delay angle to eccentric ratio ( $\alpha=30$ deg).

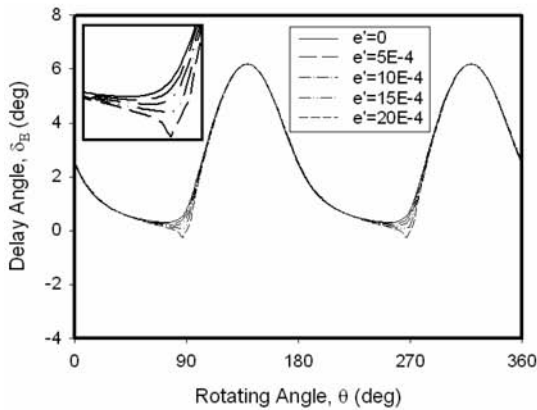


Fig. 11. Delay angle to eccentric ratio ( $\alpha=25$ deg).

angle according to the change of the eccentricity ratio when the swivel angle is 15 and 25deg, respectively. The minimum delay angle approaches zero with the increase of the eccentricity ratio of the disk, and it exhibits the same tendency as that in Fig. 9.

Fig. 12 shows the change of the delay angle according to the change of the eccentricity ratio when the swivel angle is 27deg. According to the changes in the eccentricity ratio of the disk, the critical value of the delay angle, whose minimum delay angle is nearest to zero, is displayed.

Fig. 13 shows the tendency that, if the swivel angle is above 27deg, the minimum delay angle is always less than zero regardless of the eccentricity ratio. When the minimum delay angle becomes less than zero, the piston rod changes the driving direction from along the direction of shaft rotation to the opposite direction, so it cannot drive the cylinder block normally.

Fig. 14 shows the change of the delay angle according to the change of the swivel angle when the eccentricity ratio is 15E-4. When the swivel angle is less than 27 deg, the values of the delay in the driving ranges are greater than zero, so normal driving is possible.



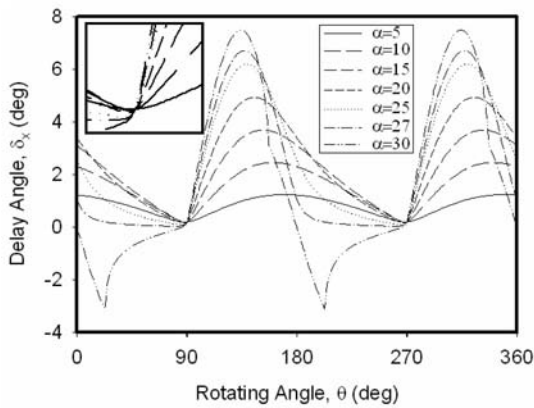


Fig. 14. Delay angle to eccentric ratio ( $e' = 15E-4$ ).

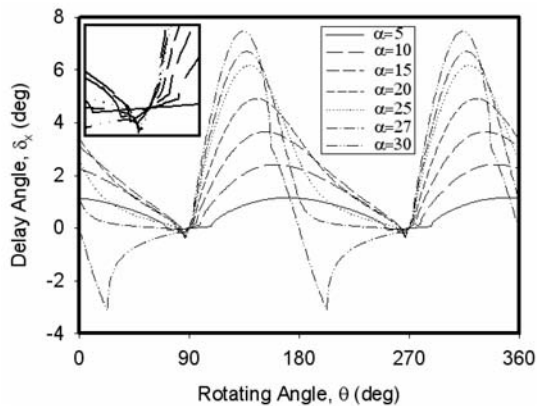


Fig. 15. Delay angle to eccentric ratio ( $e' = 20E-4$ ).

Fig. 15 shows that when the eccentricity ratio is  $20E-4$ , there exists a value which is less than zero in the driving range of the delay angle, regardless of the change of the swivel angles, and it is impossible to drive normally.

Therefore, when the taper angle  $\xi$ , the radius of the disk's spherical part  $R_d$ , the radius of the cylinder bore  $R_c$ , and the length of the piston rod  $L$  are chosen beforehand, the maximum swivel angle can be determined and the maximum eccentricity ratio can be designed.

#### 4. Conclusions

Based on the above theoretical analyses about the driving mechanism of the bent-axis type oil hydraulic piston pump driven by a piston rod, if the design parameters such as the taper angle  $\xi$ , the radius of the disk's spherical part  $R_d$ , the radius of the cylinder bore  $R_c$ , and the length of the piston rod  $L$  are

given, the following conclusions can be obtained :

- (1) In the bent-axis type oil hydraulic piston pump driven by piston rod, when the minimum value of the delay angle is greater than zero, there exist normal driving ranges that take the point of the minimum delay angle as centers in the first quadrant and the third quadrant.
- (2) In the same condition, the maximum swivel angle of the bent-axis type oil hydraulic piston pump driven by the piston rod can be determined by theoretical analysis. And then, when the swivel angle is greater than the calculated maximum value, the delay angle becomes less than zero, regardless of the eccentricity ratio, so the normal driving by the piston rod is impossible.
- (3) If the eccentricity ratio of the disk center is less than  $15E-4$  and the swivel angle is below the calculated maximum value, the minimum delay angle is always larger than zero, so normal driving is possible. However, if the eccentricity ratio is greater than  $15E-4$ , the minimum delay angle is always less than zero irrespective of the swivel angle, so normal driving is impossible.

#### Nomenclature

- $e$  : Eccentric distance between  $O_d'$  and  $O_d$   
 $e'$  : Eccentricity ratio between  $e$  and  $R_d$   
 $L$  : Length between  $P_1$  and  $P_2$  in tapered piston  
 $O_d$  : Center of driving shaft  
 $O_d'$  : Center of disk on driving shaft  
 $O_c$  : Center of cylinder block  
 $O_b$  : Center of cylinder bore  
 $P_1$  : Center of piston head  
 $P_1'$  : Ahead center of piston head  
 $P_2$  : Center of piston end  
 $P_2'$  : Behind center of piston end  
 $R_c$  : Radius between  $O_c$  and  $O_b$   
 $R_d$  : Radius between  $O_d$  and  $P_1$   
 $R_{dc}$  : Distance between  $O_b$  and projected  $P_1$  on cylinder block  
 $R_e$  : Equivalent distance between  $O_b$  and projected  $P_1$  on cylinder block  
 $R_s$  : Distance between  $O_d$  and  $P_1$   
 $\theta$  : Rotational angle of driving shaft  
 $\alpha$  : Swivel angle between cylinder block and driving shaft

- $\beta_0$  : Tilting angle ( $\angle P_1 P_2 O_b$ )  
 $\beta_c$  : Equivalent tilting angle ( $\angle P_1 P_2 O_c$ )  
 $\delta_0$  : Ahead delay angle ( $\angle P_1 O_c O_b$ )  
 $\delta_e$  : Equivalent ahead delay angle ( $\angle P_1 O_c O_b$ )  
 $\delta_x$  : Delay angle ( $\angle P_2 O_c P_2'$ )  
 $\delta_E$  : Equivalent delay angle ( $\angle P_2 O_c P_2'$ )  
 $\phi$  : Eccentric angle  
 $\phi'$  : Eccentric angle ratio  $\phi$  and  $2\pi/7$   
 $\varphi_0$  : Phase angle ( $\angle P_1 O_b O_c$ )  
 $\varphi_e$  : Equivalent phase angle ( $\angle P_1 O_b O_c$ )  
 $\xi$  : Taper angle of piston

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